LOONES
VICTOR

Banque Oral N° 23

On calcule le polynôme caractéristique

$$\chi_M = \begin{vmatrix} X-a & -b & -b & -b \\ -b & X-a & -b & -b \\ -b & -b & X-a & -b \\ -b & -b & -b & X-a \end{vmatrix}$$

$$\underset{C_1 \leftarrow C_1+C_2+C_3+C_4}{=} \begin{vmatrix} X-(a+3b) & -b & -b & -b \\ X-(a+3b) & X-a & -b & -b \\ X-(a+3b) & -b & X-a & -b \\ X-(a+3b) & -b & -b & X-a \end{vmatrix}$$

$$= (X-(a+3b)) \begin{vmatrix} 1 & -b & -b & -b \\ 1 & X-a & -b & -b \\ 1 & -b & X-a & -b \\ 1 & -b & -b & X-a \end{vmatrix}$$

$$\underset{\substack{L_2 \leftarrow L_2-L_1 \\ L_3 \leftarrow L_3-L_1 \\ L_4 \leftarrow L_4-L_1}}{=} \begin{vmatrix} 1 & -b & -b & -b \\ 0 & X-a+b & 0 & 0 \\ 0 & 0 & X-a+b & 0 \\ 0 & 0 & 0 & X-a+b \end{vmatrix} \times (X-(a+3b))$$

$$= (X-(a+3b))(X-(a-b))^3$$

$$Sp(M) = \{a+3b\ ,\ a-b\}$$

équivalences !!!

<u>Cas 1</u> $a+3b = a-b \Rightarrow 3b = -b$

$\Rightarrow b = 0$

Alors $M = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{pmatrix}$

et $\forall n \in \mathbb{N}$: $M^n = \begin{pmatrix} a^n & 0 & 0 & 0 \\ 0 & a^n & 0 & 0 \\ 0 & 0 & a^n & 0 \\ 0 & 0 & 0 & a^n \end{pmatrix}$

<u>Cas 2</u> $b \neq 0$, ie $a+3b \neq a-b$

$M - (a-b)I_n = \begin{pmatrix} b & b & b & b \\ b & b & b & b \\ b & b & b & b \\ b & b & b & b \end{pmatrix}$

$\dim(E_{(a-b)}(M)) = 3$ car...

Justifier

donc M est diagonalisable car
$\dim(E_{(a-b)}(M)) + \dim(E_{a+3b}(M)) = 3+1 = 4$

De plus $C_2 - C_1 = C_3 - C_1 = C_4 - C_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

donc $E_{(a-b)}(M) = \mathrm{Vect}\Big(\underbrace{\begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}}_{\text{bien libre...}}\Big)$

$M - (a+3b)I_n = \begin{pmatrix} -3b & b & b & b \\ b & -3b & b & b \\ b & b & -3b & b \\ b & b & b & -3b \end{pmatrix}$ $C_1 + C_2 + C_3 + C_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

donc $E_{(a+3b)}(M) = \mathrm{Vect}\left(\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}\right)$

d'où $M = PDP^{-1}$ $P = \begin{pmatrix} -1 & -1 & -1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

Une orthodiagonalisation donnerait un P^{-1} facilement calculable.

$D = \begin{pmatrix} a-b & 0 & 0 & 0 \\ 0 & a-b & 0 & 0 \\ 0 & 0 & a-b & 0 \\ 0 & 0 & 0 & a+3b \end{pmatrix}$

$M^n = PD^nP^{-1}$