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Formulaire 1 : Trigonométrie

$$\cos\theta = \mathfrak{Re}(e^{i\theta}) = \frac{e^{i\theta}+e^{-i\theta}}{2}$$
$$\sin\theta = \mathfrak{Im}(e^{i\theta}) = \frac{e^{i\theta}-e^{-i\theta}}{2i}$$

$$\sin' = \cos\,;\ \cos' = -\sin\,;\ \tan' = \frac{1}{\cos^2} = 1+\tan^2.$$

$$\cos^2+\sin^2 = 1 \text{ et } e^{i\theta} = \cos\theta + i\sin\theta.$$

$$\frac{\sin x}{x} \xrightarrow[x\to 0]{} 1 \text{ et } \frac{\tan x}{x} \xrightarrow[x\to 0]{} 1.$$

$$\sin x = \sin y \iff x \equiv y\ [2\pi] \text{ ou } x \equiv \pi - y\ [2\pi]$$

$$\cos x = \cos y \iff x \equiv y\ [2\pi] \text{ ou } x \equiv -y\ [2\pi]$$

$$\tan x = \tan y \iff x \equiv y\ [\pi]$$

$$\cos(a+b) = \cos a\cos b - \sin a \sin b$$

$$\sin(a+b) = \sin a\cos b + \sin b\cos a$$

$$\cos(a-b) = \cos a\cos b + \sin a\sin b$$

$$\sin(a-b) = \sin a\cos b - \sin b\cos a$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1-\tan a\tan b} \qquad \tan(a-b) = \frac{\tan a - \tan b}{1+\tan a\tan b}$$

$$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x = \cos^2 x - \sin^2 x$$

$$\sin 2x = 2\sin x\cos x \qquad \tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\cos^2 x = \frac{1+\cos 2x}{2} \qquad \sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos a\cos b = \frac{1}{2}\Big(\cos(a+b)+\cos(a-b)\Big)$$
$$\sin a\cos b = \frac{1}{2}\Big(\sin(a+b)+\sin(a-b)\Big)$$
$$\sin a\sin b = \frac{1}{2}\Big(\cos(a-b)-\cos(a+b)\Big)$$

$$\cos p + \cos q = 2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$
$$\sin p + \sin q = 2\sin\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$
$$\cos p - \cos q = -2\sin\left(\frac{p+q}{2}\right)\sin\left(\frac{p-q}{2}\right)$$
$$\sin p - \sin q = 2\sin\left(\frac{p-q}{2}\right)\cos\left(\frac{p+q}{2}\right)$$

On pose $t = \tan\frac{x}{2}$, alors

$$\cos x = \frac{1-t^2}{1+t^2} \quad ; \quad \sin x = \frac{2t}{1+t^2} \quad ; \quad \tan x = \frac{2t}{1-t^2}$$

$$x^2+y^2 = 1 \implies \exists\theta\in\mathbb{R},\ \ x = \cos\theta \text{ et } y = \sin\theta$$

$$\exists\varphi,\psi\in\mathbb{R},\ \ \forall\theta\in\mathbb{R},$$

$$a\cos\theta + b\sin\theta = \sqrt{a^2+b^2}\cos(\theta+\varphi) = \sqrt{a^2+b^2}\sin(\theta+\psi)$$

$$\operatorname{ch} x = \frac{e^{x}+e^{-x}}{2}$$
$$\operatorname{sh} x = \frac{e^{x}-e^{-x}}{2}$$

$$\operatorname{sh}' = \operatorname{ch}\,;\ \operatorname{ch}' = \boxed{+}\operatorname{sh}\,;\ \operatorname{th}' = \frac{1}{\operatorname{ch}^2} = 1\boxed{-}\operatorname{th}^2.$$

$$\operatorname{ch}^2\boxed{-}\operatorname{sh}^2 = 1 \text{ et } \exp = \operatorname{ch}+\operatorname{sh}.$$

$$\frac{\operatorname{sh} x}{x} \xrightarrow[x\to 0]{} 1 \text{ et } \frac{\operatorname{th} x}{x} \xrightarrow[x\to 0]{} 1.$$

$$\operatorname{sh} x = \operatorname{sh} y \iff x = y$$

$$\operatorname{ch} x = \operatorname{ch} y \iff x = y \text{ ou } x = -y$$

$$\operatorname{th} x = \operatorname{th} y \iff x = y$$

$$\operatorname{ch}(a+b) = \operatorname{ch} a\operatorname{ch} b\boxed{+}\operatorname{sh} a\operatorname{sh} b$$

$$\operatorname{sh}(a+b) = \operatorname{sh} a\operatorname{ch} b + \operatorname{sh} b\operatorname{ch} a$$

$$\operatorname{ch}(a-b) = \operatorname{ch} a\operatorname{ch} b\boxed{-}\operatorname{sh} a\operatorname{sh} b$$

$$\operatorname{sh}(a-b) = \operatorname{sh} a\operatorname{ch} b - \operatorname{sh} b\operatorname{ch} a$$

$$\operatorname{th}(a+b) = \frac{\operatorname{th} a + \operatorname{th} b}{1\boxed{+}\operatorname{th} a\operatorname{th} b} \qquad \operatorname{th}(a-b) = \frac{\operatorname{th} a - \operatorname{th} b}{1\boxed{-}\operatorname{th} a\operatorname{th} b}$$

$$\operatorname{ch} 2x = 2\operatorname{ch}^2 x - 1 = 1\boxed{+}2\operatorname{sh}^2 x = \operatorname{ch}^2 x\boxed{+}\operatorname{sh}^2 x$$

$$\operatorname{sh} 2x = 2\operatorname{sh} x\operatorname{ch} x \qquad \operatorname{th} 2x = \frac{2\operatorname{th} x}{1\boxed{+}\operatorname{th}^2 x}$$

$$\operatorname{ch}^2 x = \frac{1+\operatorname{ch} 2x}{2} \qquad \operatorname{sh}^2 x = \frac{\boxed{\operatorname{ch} 2x - 1}}{2}$$

$$\operatorname{ch} a\operatorname{ch} b = \frac{1}{2}\Big(\operatorname{ch}(a+b)+\operatorname{ch}(a-b)\Big)$$
$$\operatorname{sh} a\operatorname{ch} b = \frac{1}{2}\Big(\operatorname{sh}(a+b)+\operatorname{sh}(a-b)\Big)$$
$$\operatorname{sh} a\operatorname{sh} b = \frac{1}{2}\Big(\boxed{\operatorname{ch}(a+b)-\operatorname{ch}(a-b)}\Big)$$

$$\operatorname{ch} p + \operatorname{ch} q = 2\operatorname{ch}\left(\frac{p+q}{2}\right)\operatorname{ch}\left(\frac{p-q}{2}\right)$$
$$\operatorname{sh} p + \operatorname{sh} q = 2\operatorname{sh}\left(\frac{p+q}{2}\right)\operatorname{ch}\left(\frac{p-q}{2}\right)$$
$$\operatorname{ch} p - \operatorname{ch} q = \boxed{+}2\operatorname{sh}\left(\frac{p+q}{2}\right)\operatorname{sh}\left(\frac{p-q}{2}\right)$$
$$\operatorname{sh} p - \operatorname{sh} q = 2\operatorname{sh}\left(\frac{p-q}{2}\right)\operatorname{ch}\left(\frac{p+q}{2}\right)$$

On pose $t = \operatorname{th}\frac{x}{2}$, alors

$$\operatorname{ch} x = \frac{1\boxed{+}t^2}{1\boxed{-}t^2} \quad ; \quad \operatorname{sh} x = \frac{2t}{1\boxed{-}t^2} \quad ; \quad \operatorname{th} x = \frac{2t}{1\boxed{+}t^2}$$